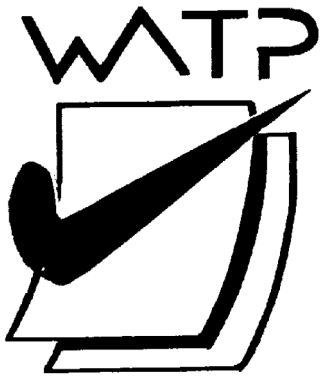


Semester Two Examination 2016

Question/Answer Booklet

MATHEMATICS METHODS UNITS 3 & 4

Section Two: Calculator-assumed

Student Name: _____

Teacher's Name: _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for paper: one hundred minutes

Material required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens(blue/black preferred), pencils(including coloured), sharpener, correction tape/fluid, erasers, ruler, highlighters

Special Items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations.

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

	Number of questions available	Number of questions to be attempted	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	8	8	50	50	35
Section Two Calculator—assumed	**13**	**13**	**100**	**100**	**65**
					100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2016.* Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions according to the following instructions.

   Section Two: Write answers in this Question/Answer Booklet. Answer **all** questions.

   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

   It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

See next page

SectionTwo: Calculator–assumed 100 marks

This section has **fourteen (14)** questions. Attempt **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time: 100 minutes

Question 9 (8 marks)

(a) Determine the value of

(i) $\lim_{h \to \infty} \left(1 + \frac{1}{3h}\right)^h$ (2 marks)

(ii) $\left(\lim_{t \to 0} \left(\frac{1 - e^t}{t}\right)\right)^2$ (2 marks)

(b) A population of bacteria is known to increase in size by 50% every 2 hours. There are 2000 bacteria in the population at 12 noon.

Determine:

(i) the constant of proportionality, correct to four decimal places. (2 marks)

(ii) the time when the population will reach 8000. (2 marks)

See next page

Question 10 (6 marks)

A particle is moving in rectilinear motion such that the acceleration a m/s^2, is given as $a = -\pi^2 \sin \pi t$. Initially the particle is stationary at the origin.

(a) Show clearly that the displacement is given as $x = \sin \pi t - \pi t$. (4 marks)

(b) Determine:

(i) the velocity of the particle when $t = 2$. (1 mark)

(ii) the maximum acceleration of the particle. (1 mark)

See next page

Question 11 (9 marks)

(a) A spherical balloon is being inflated.
Use the Incremental Change Formula to determine the approximate change in the volume of the balloon when the radius of the balloon increases by 30%. (4 marks)

(b) Two particles, A and B, are moving simultaneously on the same line.
Particle A is moving with velocity given as $v_A = 3t^2 - 4t + 3$, while particle B is moving such that its displacement is given as $x_B = 2 - 3t - t^3$.
Particle A begins its motion from the origin.

(i) Show that the distance between the two particles, at any time t is given by $2t^3 - 2t^2 + 6t - 2$. (3 marks)

(ii) When *and* where are the particles at the same displacement? (2 marks)

See next page

Question 12 (8 marks)

The probability function $f(x)$ is defined as:

$$f(x) = \begin{cases} ax^2 + 1 & 0 \leq x \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the value of a. (3 marks)

(b) Calculate the following probabilities.

(i) $P\left(X < \frac{1}{4}\right)$ (2 marks)

(ii) $P\left(X < \frac{1}{8} \mid X < \frac{1}{4}\right)$ (2 marks)

A second function $g(x)$ is defined as:

$$g(x) = \begin{cases} 1 - x & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(c) Explain why $g(x)$ cannot be a pdf. (1 mark)

See next page

Question 13 (10 marks)

The rate of change in area of an algal–bloom infestation in a local part of the river is modelled by

$\frac{dA}{dt} = -t^2 + e^{0{\cdot}4t}$, where A is the area in m^2 infested, at any time t, days from June 1st.
On June 1st there was 6000 m^2 infested.

(a) Determine:

(i) the rate of change in area on June 5th, ie. when $t = 4$, correct to the nearest m^2.(2 marks)

(ii) the date, when the rate of change in the area infested is a minimum. (3 marks)

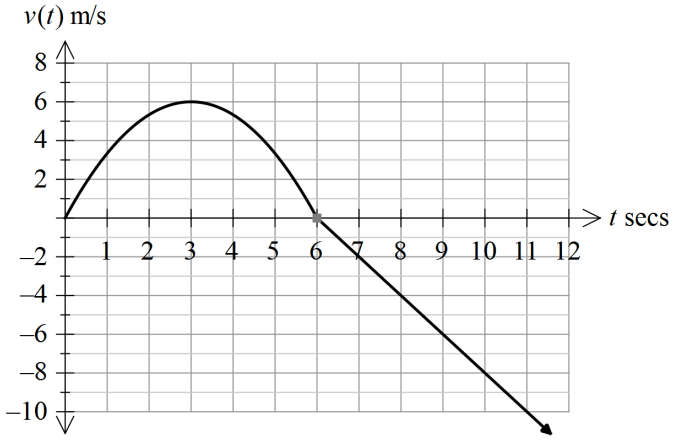
(b) What is the total change in area infested between June 1st and June 12th inclusive? (3 marks)

(c) What is the total area infested by the end of June 15th, to the nearest m^2? (2 marks)

See next page

Question 14 (7 marks)

A particle undergoes rectilinear motion, with its $v - t$ graph shown below.



(a) Determine the equation of velocity for the:

(i) first six seconds. (2 marks)

(ii) remaining time in motion. (1 mark)

(b) How far had the particle travelled in the first six seconds? (2 marks)

(c) At what time will the particle return to its original position? (2 marks)

See next page

Question 15 (8 marks)

A new medicine is being developed using drug trials on ten patients. Previous trials have established that there is a 45% success rate.

(a) Find the mean and standard deviation of the Binomial distribution. (2 marks)

(b) Determine the probability that:

(i) exactly 5 patients have a successful trial. (2 marks)

(ii) at most 6 patients have a successful trial. (2 marks)

(iii) at least 7 patients do not have a successful trial, given that at most 6 patients have a successful trial. (2 marks)

See next page

Question 16 (4 marks)

The probability that a customer has to wait between a and b minutes at a fast food outlet is modelled by:

$$P(a \leq t \leq b) = \int_a^b 0{\cdot}25e^{-0{\cdot}25t} \quad \text{where } t \geq 0$$

(a) Determine the probability that the next customer has to wait between 1 and 3 minutes more for their meal, given they have already waited more than 3 minutes for their meal. (2 marks)

(b) What is the median waiting time for a meal at the outlet? (2 marks)

See next page

Question 17 (9 marks)

The heights of 150 Year 12 girls at the local high school are normally distributed with a mean of 162 cm and a standard deviation of 5 cm.

(a) Determine:

(i) the probability that a randomly selected girl is less than 164 cm tall. (2 marks)

(ii) the number of girls who are between 161 and 163 cm tall, given that they are less than 164 cm tall. (2 marks)

The shortest ten percent of girls were classified as being too short to be goal shooters in their netball side.

(b) What is the height of the shortest girl who would be classified as a goal shooter? (2 marks)

The heights of 120 Year 12 boys at the same school were also normally distributed with a mean height of 175 cm. 40 of these boys were over 180 cm tall.

(c) Find the standard deviation of the heights of the boys. (3 marks)

See next page

Question 18 (8 marks)

An investigation into left-handedness amongst cricketers in WA grade cricket clubs was conducted.
A sample of 59 cricketers was randomly selected.
Of these, 15 were found to be left handed.

(a) Determine $\hat{p}$, the sample proportion, correct to 4 decimal places. (1 mark)

The population mean for "left-handedness" is p.

(b) Determine a 95% confidence interval for p based on the sample of 59. (3 marks)

(c) What size is the margin of error in this inference? (1 mark)

(d) How could the margin of error be decreased without altering the confidence level? (1 mark)

(e) What sample size would be required to make the margin of error $\pm$ 4%? (2 marks)

See next page

Question 19 (8 marks)

A discrete random variable has a pdf defined below, with an expected value given by E(x) = 32.

x	10	20	30	40	50
$P(X = x)$	0.1	0.2	m	n	0.2

(a) Determine:

(i) the values of m and n. (2 marks)

(ii) $P(X > 40 \cup X < 20)$ (1 mark)

(iii) $P(X > 30 \mid X > 20)$ (2 marks)

All values of X are divided by 10 and then 10 is added.

(b) For this new distribution state:

(i) the expected value. (1 mark)

(ii) the variance. (2 marks)

See next page

Question 20 (9 marks)

A Mathematics test is used to compare Year 4 students throughout the State.
8000 students sit for the test each year and are classified as competent or not.

This year a random sample of 90 students was tested, with 54 being classified as competent.

(a) What is meant by a random sample? (1 mark)

(b) Calculate a point estimate for the mean. (1 mark)

(c) Determine a 90% confidence interval for the population proportion.
Show working, including the margin of error. (4 marks)

The test was administered to two other random samples.

(d) Use the 90% confidence interval determined for the proportion of Year 4 students to comment on the following random samples.

(i) 50 students were tested and 35 were classified as competent. (1 mark)

(ii) 120 students were tested and 71 were classified as competent. (2 marks)

See next page

Question 21 (6 marks)

(a) If $\int_0^e \frac{ax}{x^2 + e^2}\, dx = \ln 2$, determine the value of the constant a, by using Calculus methods. (4 marks)

(b) Channel 2 News conducted a phone poll with responses to the question:
"Should the drinking age be raised to 21 in Australia?"

Comment on this method of random sampling and discuss the possible sources of bias.
(2 marks)

END OF QUESTIONS

Additional working space

Question number(s): ……